Making Sense of Representativeness

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Plan

1. What is representativeness?
2. Irrational?
3. Making sense of representativeness

1.
What is representativeness?

(1972). Subjective Probability: A Judgment of Representativeness

(1973). On the Psychology of Prediction.

Example: Lawyers & Engineers

Example: Lawyers & Engineers

Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers. Now, Jones is given the following description:

Scenario 1:

> *(D_1) Jack is a 45-year-old man. He is married and has four children. He is generally conservative, careful, and ambitious. He shows no interest in political and social issues and spends most of his free time on his many hobbies which include home carpentry, sailing, and mathematical puzzles. (Kahneman & Tversky 1973, p. 241)*

- In line with Kahneman and Tversky's subjects, Jones would reply:

  (E_1) *It is 55% likely that Jack is an engineer*.

Example: Lawyers & Engineers

Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers. Now, Jones is given the following description:

Scenario 2:

> *(D_2) Dick is a 30-year-old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues. (Kahneman & Tversky 1973, p. 241)*

- In line with Kahneman and Tversky's subjects, Jones would reply:

  (E_2) *It is 50% likely that Jack is an engineer*.

Example: Lawyers & Engineers

- Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers.

Scenario 3 (null description):

Jones is asked the following:

> *(D_3)Suppose an individual is chosen at random from this population. What do you think the probability is of this individual being an engineer?*

- In line with Kahneman and Tversky's subjects, Jones would reply:

> (E_3) *It is 30% likely that Jack is an engineer*.

- E_i represents the hypothesis Jones is making when inferring from the prior probabilities and the new evidence given in Jack's and Dick's description
- D_i represents the description Jones is given each time
- So, what experimenters asked was for subjects to make a prediction based on:
- Base rates: P(Eng) = 0.3 and P(Law) = 0.7
- Some evidence: D_i
- So, what Jones ought to have done is to have used Bayes' Theorem:

$$P(E_i|D_i) = \frac{P(D_i|Eng)P(Eng)}{P(D_i)} = \frac{P(D_i|Eng)P(Eng)}{P(D_i|Eng) + P(D_i|\neg Eng)}$$

Example: Lawyers & Engineers

Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers. Now, Jones is given the following description:

Scenario 1: | Evidence is stereotypical of an engineer

> *(D_1) Jack is a 45-year-old man. He is married and has four children. He is generally conservative, careful, and ambitious. He shows no interest in political and social issues and spends most of his free time on his many hobbies which include home carpentry, sailing, and mathematical puzzles. (Kahneman & Tversky 1973, p. 241)*

- In line with Kahneman and Tversky's subjects, Jones would reply:

  (E_1) *It is 55% likely that Jack is an engineer*.

Example: Lawyers & Engineers

Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers. Now, Jones is given the following description:

Scenario 2: evidence is individualized but non-informative about Dick's profession

> *(D_2) Dick is a 30-year-old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues. (Kahneman & Tversky 1973, p. 241)*

- In line with Kahneman and Tversky's subjects, Jones would reply:

  (E_2) *It is 50% likely that Jack is an engineer*.

Example: Lawyers & Engineers

- Jones is presented with information about proportions of lawyers and engineers in a population of 100 people. The proportion is 70 lawyers to 30 engineers.

Scenario 3 (null description): evidence is not individualized

Jones is asked the following:

> *(D_3)Suppose an individual is chosen at random from this population. What do you think the probability is of this individual being an engineer?*

- In line with Kahneman and Tversky's subjects, Jones would reply:

> (E_3) *It is 30% likely that Jack is an engineer*.

Summary of the findings

Scenario 1 (stereotypical)	Scenario 2 (ind. non-info)	Scenario 3 (null)
55 %	50 %	30 %

E_1: *It is 55% likely that Jack is an engineer.*

E_2: *It is 50% likely that Jack is an engineer*.

E_3: *It is 30% likely that Jack is an engineer.*

Summary of the findings

- If you tell Jones that there is this nameless person chosen at random, Jones uses base rates to calculate the posterior probabilities.
- If you tell Jones that there is this person named Dick and give no useful information about Dick's profession, Jones is going to reject the base rates and is going to go for something near 0.5 credence.
- If you give Jones some individualized description of a stereotypical engineer (doesn't have interest in politics and likes mathematical puzzles), she is going again to neglect base rates to give the estimates.
- People *don't seem to follow the correct rule* to estimate whether the evidence supports, and to which extent, certain hypothesis

Question

If people are not following Bayes' Theorem in cases like lawyers/engineers, what are they doing?

Answer

They are using a judgement of representativeness to come up with their subjective probabilities!

- Representativeness is the reasoning strategy used by Jones when answering that *It is x likely that Jack/Dick is an engineer*.
- *E* represents the hypothesis Jones is making when inferring from the prior probabilities and the new evidence given in Jack’s description.

- Thus, representativeness is an *inductive reasoning strategy* used to answer the question: *What it is to be a good example of a category?*
- In our example, Jack is a good example of the category of engineers because Jack is *representative* of engineers
- The way representativeness is classically cashed out is in terms of similarity
- Roughly, how good an example is of a category or a process is determined by *how similar* this example is with respect to a mental model or schema or stereotype people have of the category or the process.

2.
Irrational?

Is this irrationality?

- Kahneman and Tversky themselves are cautious of talking about irrationality.
- People in subsequent literature, though, argue that cases of representativeness constitute examples of human irrationality (*cf.* Cohen 1981).
- In short, the argument is that people should update using conditionalization, because otherwise they would be Dutch Book vulnerable

Is this irrationality?

- Importantly, Kahneman and Tversky showed that if you change the base rates and show the same descriptions to a Low Engineer group and to a High Engineer group, subjects would systematically deviate from the Bayesian prediction

  Low Engineer group: 30 engineers - 70 lawyers

  High Engineer group: 70 engineers - 30 lawyers

Is this irrationality?

- A case can be constructed from here to show the Dutch Book vulnerability of Jones in Scenario 1 and Scenario 2.
- Jones' estimate was not supported by the evidence, so there's a case to be made for her epistemic irrationality
- Jones would incur in a series of bets that would lead to sure losses.
- Usually, Dutch book vulnerability shows some inconsistency in agents' attitudes (see Pettigrew 2020).
- Pettigrew (2016) makes a case for a similar case involving probabilistic estimates, namely cases of the conjunction fallacy (Linda cases).

Summary thus far

- People systematically neglect base rates when given a task of prediction of a person’s profession based merely on a description.
- Not only people neglect base rates, but they do even when base rates are made salient in the description.
- People use base rates when they are given no specific information about the person’s profession.
- Representativeness is making use of a similarity judgement instead of making use of some Bayesian prediction.
- In subsequent literature, these findings have been taking as demonstrating some kind of irrationality.
- A case for the *epistemic* irrationality of this tendency can be made using Dutch Book arguments.

3.

Can we make sense of representativeness?

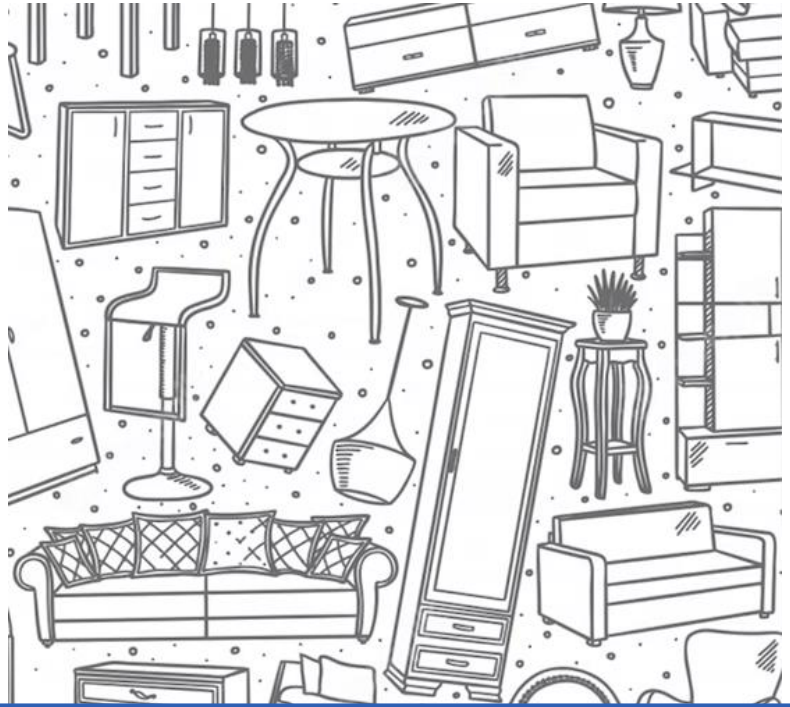
What I want to argue

- There is a missing part in the discussion of norms of updating:

  Guidance-giving notion ≠ Evaluative notion

- When we seek norms for how to go about our lives (recipes, directions on a map, Ikea manuals), we are looking for guidance-giving prompts that fit our current state of understanding of the issue.

Consider: Dinning table



Consider: Dinning table

- Consider: Smith is performing some task. Particularly, she is putting together an Ikea dining table. She is not very good at spatial reasoning, so she constantly puts right-hand-side screws in the left-hand-side of the table. She only notices later in the process.
- The guidance-giving prompts Smith is following is not suited to her abilities.
- Take away 1, Agent-relativity: Although the generic instruction manuals guarantee the same *results*, experience from Smith shows that these instruction manuals are not optimal *regardless* the user is and what limitations the it has

Consider: Dinning table

- Take away 1, **Agent-relativity:** Although the generic instruction manuals guarantee the same *results*, experience from Smith shows that these instruction manuals are not optimal *regardless* the user is and what limitations the it has.
- Take away 2, **Rational errors:** Some mistakes should be expected from applying guidance-giving rules that don’t take into account people’s cognitive limitations.

- The analogy I want to make is between instructions in general used as guides and as standards of evaluation and the notion of epistemic rationality encoded in Bayes' Theorem for the case of representativeness.
- If we don't distinguish between different functions of Bayes' Theorem, we may have different verdicts depending on whether subjects have been tutored or have corroborated the results of their estimates.
- This is compatible with the idea that some norms that guide us *also* help us evaluate the results of our performances.
- This means that even if we only care about truth or accuracy, the norms for forming certain attitudes can be different from the *optimal* norms to get to the truth.
- Why? Because in everyday life, given our cognitive limitations, we form attitudes in ways that are not accuracy-*optimal*. (See Tang 2015, Staffel 2019)

- My question: Is it possible that guidance-giving norms are epistemically rational?
- My answer: It depends on contingencies like whether we are already knowledgeable in the subject matter or whether the representation of information trigger heuristics.
- This leaves room for optimal notions of the norms, like Bayes' Theorem, but doesn't lead to the verdict that human beings are irrational when they fail to make Bayesian updates.
- Optimal notions aren't there primarily for guidance, but for the *evaluation* of our attitudes.
- To sum up: guidance-giving norms serve the purpose of helping us in the *process* of forming an attitude, while evaluative norms serve the purpose of evaluating the *results* of these processes.
- This result allows to understand why epistemically rational capacities are still in play in cases where agents can be criticized for making mistakes.

- Some evidence help substantiate my claim that representation of the problem can have an influence over which rules we use in reasoning.
- Without endorsing the whole program, in Gigerenzer's tradition, representativeness has been shown to be sensitive to certain epistemic capacities, like the representation of the cognitive tasks. (Gigerenzer et al., 1988)
- Instead of presenting a verbal description, they presented a problem with the same structure (base rates, stereotypical descriptions, requirement to make a probabilistic estimate or prediction) but represented graphically (Sedlmeier & Gigerenzer, 2001).
- Random sampling was visually observable for experimental subjects and this brought their estimates closer to the Bayesian results. (See here for an example)

- **Lawyers/Engineers*:** Jones have already made the estimate that it is 0.55 likely that Jack is an engineer. Now, thanks to co-participant subject in the experiment Jones learns that later on the experimenter will come and give her reason to doubt about the estimate. But she also learns that the way descriptions are given in the experiment make it particularly hard to determine roughly the correct estimate. What should you do?
- Here, there's a parallel to cases from the Higher-Order Evidence literature (Christensen 2010, 2016) and from the disagreement literature. Cases where people are under the influence of a reasoning-distorting drug have a similar structure.
- Specifically, cases like this seem to recommend to Jones thinking it likely that she reasoned incorrectly from her available evidence and that ought to suspend judgement.
- Both Drug cases and Lawyers/Engineers* show that rational probabilistic estimates can depend on limitations the agent has in each case.

Wrapping up

- Judgement by representativeness may involve cognitive representations of the information that bear upon the epistemic rationality of one’s credal attitudes.
- Specifically, it seems that verbal representation of problems that require Bayesian predictive judgements make it particularly difficult for us to elicit Bayesian reasoning.
- However, if graphical representations are in place, we may approximate to Bayesian predictive judgements.
- Representativeness is not necessarily irrational, since the cognitive (representational factors) that may elicit the heuristic bear upon considerations of epistemic rationality for agents with cognitive limitations.

So there we have it…

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Thank you!

Say hi to sesanchezma@pm.me

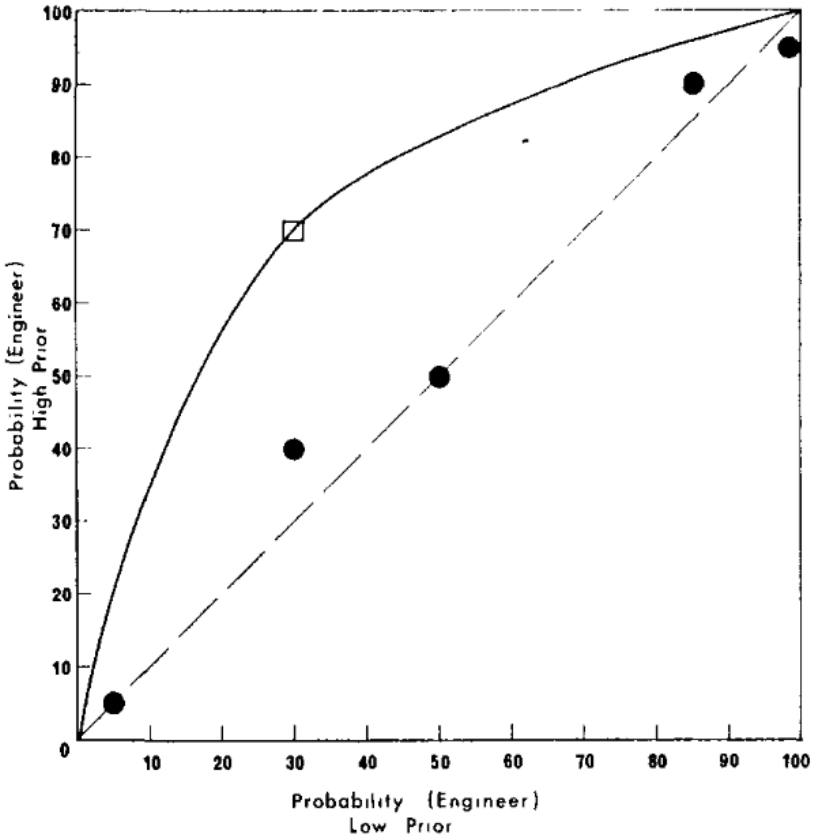
Bonus Slides…

A similar (but different) view

- It could be argued that representativeness cannot be epistemically rational, but is all-things-considered rational.
- Why? It could be argued that representativeness favors saving time and resources that are vital for our survival.
- So, being able to make quick judgements based on similarity favors an organism over time even if it gets things wrong in some instances.
- In the tradition of Herbert Simon and Gerd Gigerenzer, people usually reject the rational constraints from which people deviate.
- Their argument depends on a different account of rationality and I want my focus to be rather on *epistemic* rationality.

Kahneman & Tversky (1973)

- Median responses of subjects for five descriptions and for the null description (the square).
- The median responses are systematically deviated from the curved line, the Bayesian prediction.



Similarity of process

On each round of a game, 20 marbles are distributed at random among five children: Alan, Ben, Carl, Dan, and Ed. Consider the following distributions:

Type I		Type II	
Alan	4	Alan	4
Ben	4	Ben	4
Carl	5	Carl	4
Dan	4	Dan	4
Ed	3	Ed	4

Question:

In many rounds of the game, will there be more results of type I or of type II?

- Distribution II is more probable than distribution I (as per the formula for combinations)
- Distribution I is generally perceived as more probable, because it is more *similar* to the randomness of the distribution than the second, which gives a sense of uniformity

See for example Weng Hong Tang (2015)

- Tang is engaged in a debate about whether accounts of belief in terms of dispositions to take things for granted can account for cases when people believe some *p* but also believe that there is some small not-*p* chance
- Tang appeals, not to a disposition to take things for granted, but to a heuristic:

> *when certain probability values are close to 1 (or 0), we're disposed to employ the heuristic of reasoning as if the values are 1 (or 0), whether the probabilities concerned are subjective probabilities (i.e., credences) or chances.* (8)

- In this paper, Tang shows how it can make sense to take heuristics as the way to go when cognitive limitations are taken into account. Although his main target is to reject specific views of outright belief, his view can be seen as a delineation of an argument of how can it make sense to appeal to heuristics.

See for example Julia Staffel (2019)

- Staffel argues for a rejection of modeling cases of simplified reasoning as cases in which some type of conditionalization is involved.
- Drawing from empirical literature, Staffel shows that pseudo-conditionalization strategies are unfeasible for limited agents like human beings.
- In the end, outright beliefs in addition to credence play important roles in simplifying reasoning just as a heuristic does.
- Although using heuristics may lead to some incoherence, it is important *how much incoherence* it leads to. This sacrifice might be worth it in some simplification tasks. Cognitive limitations make tradeoffs something to be expected.
- There is a difference between epistemic constraints and non-epistemic ones: “if one endorses the heuristic view of the function of outright belief, one is thereby committed to thinking that suitable norms of belief must meet both epistemic constraints *and* feasibility constraints.” (20)